

하단은 저희 유니버싱정보출판(Tel:054-776-0881)이 수행한
수식 작업의 일부입니다. 수학, 화학, 물리학, 공학 등의 강의자료
등의 전산화가 필요하신 경우, 필사본을 저희에게 맡겨주시면
하단과 같이 세심한 작업을 해드립니다. 최대 월 10,000 페이지
이상도 작업이 가능합니다. 더하여 음성기록문자화 작업도
전문으로 하고 있습니다. 많은 관심과 연락바랍니다.

▶ 원본의 우편 및 택배발송 : TEL. 019-685-0882, 054-776-
0881 경북 경주시 동천동 880-1 한일아트빌라 102 호 임병덕
우편번호: 780-933

Now, let's consider $Q \angle R$ test statistic.

Note that

$$Q_n(\theta) = Q_n(\dot{\theta}) + \nabla_{\theta} Q_n(\dot{\theta})(\theta - \dot{\theta}) + \frac{1}{2}(\theta - \dot{\theta})' \nabla_{\theta}^2 Q_n(\ddot{\theta})(\theta - \dot{\theta})$$

For some $\ddot{\theta}$ between θ and $\dot{\theta}$,

Now, let $\dot{\theta}$ be $\hat{\theta}_n$ and θ be $\tilde{\theta}_n$.

$$\text{Then } Q_n(\tilde{\theta}_n) = Q_n(\hat{\theta}_n) + \nabla_{\theta} Q_n(\hat{\theta}_n)(\tilde{\theta}_n - \hat{\theta}_n) + \frac{1}{2}(\tilde{\theta}_n - \hat{\theta}_n)' \nabla_{\theta}^2 Q_n(\bar{\theta})(\tilde{\theta}_n - \hat{\theta}_n)$$

For some $\bar{\theta}_n$ between $\tilde{\theta}_n$ and $\hat{\theta}_n$.

Further, we note that from the definition of $\hat{\theta}_n, \nabla_{\theta} Q_n(\hat{\theta}_n) \equiv 0$. This implies that

$$\begin{aligned} 2(Q_n(\tilde{\theta}_n) - Q_n(\hat{\theta}_n)) &= (\tilde{\theta}_n - \hat{\theta}_n)' \nabla_{\theta}^2 Q_n(\bar{\theta})(\tilde{\theta}_n - \hat{\theta}_n) \\ &= \sqrt{n}(\tilde{\theta}_n - \hat{\theta}_n)' \{n^{-1} \nabla_{\theta}^2 Q_n(\bar{\theta}_n)\} \sqrt{n}(\tilde{\theta}_n - \hat{\theta}_n) \end{aligned}$$

We have already been that

$$\begin{aligned} \sqrt{n}(\hat{\theta}_n - \theta_*) &= - \left\{ n^{-1} \nabla_{\theta}^2 Q_n(\ddot{\theta}) \right\}^{-1} n^{-1/2} \nabla_{\theta} Q_n(\theta_*) \\ &= - \left\{ n^{-1} \nabla_{\theta}^2 Q_n(\theta_*) \right\}^{-1} n^{-1/2} \nabla_{\theta} Q_n(\theta_*) + o_p(1). \end{aligned}$$

And

$$\begin{aligned} \begin{bmatrix} \tilde{\theta}_n - \theta_* \\ \tilde{\lambda}_n \end{bmatrix} &= \begin{bmatrix} \nabla_{\theta}^2 Q_n(\bar{\theta}_n) & -IR' \\ -IR & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\nabla_{\theta} Q_n(\theta_*) \\ IR\theta_* - \pi \end{bmatrix} \\ \therefore \tilde{\theta}_n - \theta_* &= - \left(\nabla_{\theta}^2 Q_n(\bar{\theta}_n) \right)^{-1} \left(I + (-IR')E^{-1}(-IR) \cdot \left(\nabla_{\theta}^2 Q_n(\bar{\theta}_n) \right)^{-1} \right) \nabla_{\theta} Q_n(\theta_*) \\ &\quad + \left(\nabla_{\theta}^2 Q_n(\bar{\theta}_n) \right)^{-1} (-R')E^{-1} \cdot (IR\theta_* - \pi) \\ \therefore \sqrt{n}(\tilde{\theta}_n - \theta_*) &= - \left(n^{-1} \nabla_{\theta}^2 Q_n(\theta_*) \right)^{-1} \left(I + (-R') \cdot (n^{-1}E)^{-1} (-R) \left(n^{-1} \nabla_{\theta}^2 Q_n(\bar{\theta}_n) \right)^{-1} \right) n^{-1/2} \nabla_{\theta} Q_n(\theta_*)_{\text{th}} \\ &\quad + \left(n^{-1} \nabla_{\theta}^2 Q_n(\theta_*) \right)^{-1} (-R') (nR') (nE)^{-1} \sqrt{n} (IR\theta_* - \pi) + o_p(1) \end{aligned}$$

us

$$\begin{aligned}
\sqrt{n}(\tilde{\theta}_n - \theta_*) &= -\left(n^{-1}\nabla_{\theta}^2 Q_n(\theta_*)\right)^{-1} n^{-1/2}\nabla_{\theta} Q_n(\theta_*) \\
&\quad -\left(n^{-1}\nabla_{\theta}^2 Q_n(\theta_*)\right)^{-1}(-R')(nE)^{-1}(-IR)\left(n^{-1}\nabla_{\theta}^2 Q_n(\theta_*)\right)^{-1} n^{-1/2}\nabla_{\theta} Q_n(\theta_*) \\
&\quad +\left(n^{-1}\nabla_{\theta}^2 Q_n(\theta_*)\right)^{-1}(-IR')(nE)^{-1}\sqrt{n}(IR\theta_* - \pi) + o_p(1) \\
&= \sqrt{n}(\hat{\theta}_n - \theta_*) \\
&\quad -\left(n^{-1}\nabla_{\theta}^2 Q_n(\theta_*)\right)^{-1}(-R')(nE)^{-1}(-IR)\left(n^{-1}\nabla_{\theta}^2 Q_n(\theta_*)\right)^{-1} n^{-1/2}\nabla_{\theta} Q_n(\theta_*) \\
&\quad +\left(n^{-1}\nabla_{\theta}^2 Q_n(\theta_*)\right)^{-1}(-IR')(nE)^{-1}\sqrt{n}(IR\theta_* - \pi) + o_p(1)
\end{aligned}$$

Implying that

Digitalization

off diagonal element

diagonal element

Diagonal matrix

j square matrix having all off diagonal elements equal to zero.

$$\begin{pmatrix} d_1 & & & \circ \\ & d_2 & & \\ & & d_3 & \\ \circ & & & d_4 \end{pmatrix}$$

Theover 8.3

$$D = \begin{pmatrix} d_1 & & & \circ \\ & d_2 & & \\ & & \dots & \\ \circ & & & d_n \end{pmatrix}$$

$$\omega = \begin{pmatrix} \omega_1 & & & \circ \\ & \omega_2 & & \\ & & \omega_3 & \\ \circ & & & \omega_4 \\ & & & & \omega_n \end{pmatrix}$$

$$1. D\omega = \omega D = \begin{pmatrix} d_1\omega_1 & & & \circ \\ d_2\omega_2 & & & \\ & \dots & & \\ \circ & & & d_n\omega_n \end{pmatrix}$$

$$2. |D| = d_1 d_2 \dots d_n$$

3. D is non-singular if any diagonal element is zero.

$$(\text{ie } |D| = 0)$$

4. if D is non-singular,

$$D^{-1} = \begin{pmatrix} 1/d_1 & & & \circ \\ & 1/d_2 & & \\ & & \dots & \\ \circ & & & 1/d_n \end{pmatrix}$$

5. $\lambda_i = \delta_i$, (i.e., Eigen values of D are it's main diagonal elements)

Example) 8.6

$$x_1(0)=2$$

$$x_2(0)=1$$

$$x' = Ax + \begin{pmatrix} e^t \\ \sin t \end{pmatrix} \quad A = \begin{pmatrix} -1 & 4 \\ 0 & 3 \end{pmatrix}$$

$$\text{homogeneous soln} \quad p = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$X_h = P \Omega_D(t) C = \Omega(t) C \quad D = \begin{pmatrix} -1 & \\ & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{3t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t} & e^{3t} \\ 0 & e^{3t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} c_1 e^{-t} + c_2 e^{3t} \\ 0 + c_2 e^{3t} \end{pmatrix} \Rightarrow \begin{pmatrix} c_1 + c_2 = 2 \\ c_2 = 1 \end{pmatrix}$$

$$\therefore X_h = \begin{pmatrix} e^{-t} & e^{3t} \\ 0 & e^{3t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\parallel \quad \parallel$
 $\Omega \quad C$

$$X_p = \Omega(t) U(t) = \int \frac{1}{e^{2t}} \begin{pmatrix} e^{3t} & -e^{t3t} \\ e^{2t} & e^{-t} \end{pmatrix} \begin{pmatrix} et \\ \sin t \end{pmatrix} dt$$

$$U = \int \Omega^{-1} G(t) = \int \begin{pmatrix} e^{-t} & e^{3t} \\ 0 & e^{3t} \end{pmatrix} \begin{pmatrix} e^t \\ \sin t \end{pmatrix} dt$$

$$= \int \begin{pmatrix} e^{2t} & -\sin te^t \\ \sin te^{-3t} & \end{pmatrix} dt = \int \begin{pmatrix} e^t & -e^t \\ 0 & e^{-3t} \end{pmatrix} \begin{pmatrix} e^t \\ \sin t \end{pmatrix} dt$$

$$\int e^t \sin t dt = \frac{e^t (st - dt)}{2}$$

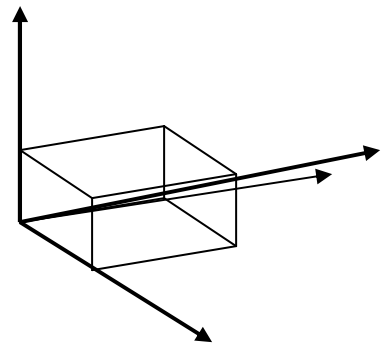
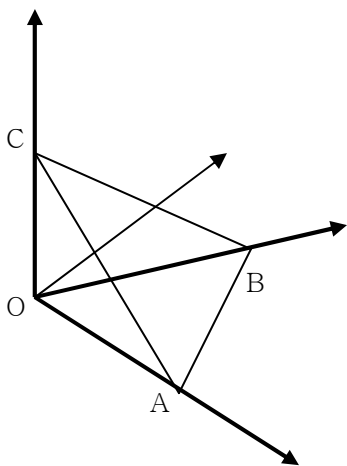
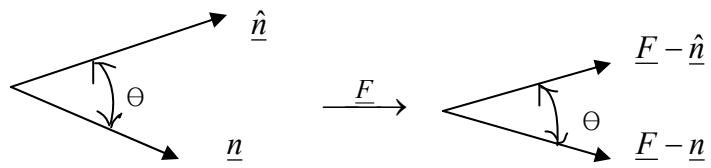
$$\int \sin te^{-3t} dt = ?$$

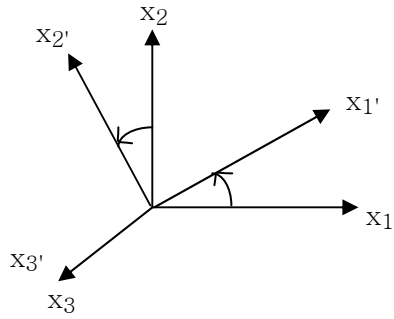
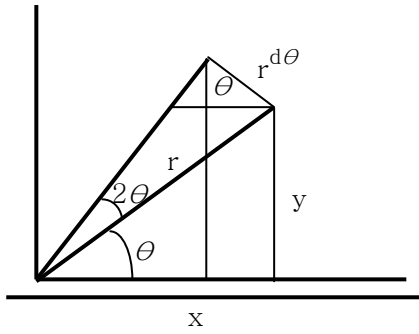
$$(\sin te^{-3t})' = -3 \sin te^{-3t} + \cos te^{-3t} \times 3$$

$$(\cos te^{-3t})' = -\sin te^{-3t} - 3 \cos te^{-3t} \times (H)$$

$$3(\sin te^{-3t})' + (\cos te^{-3t})' = -10 \sin te^{-3t}$$

$$\therefore \int \sin te^{-3t} dt = \frac{-3 \sin te^{-3t} - \cos te^{-3t}}{10}$$





$$\begin{bmatrix} \varepsilon_{11}' & \varepsilon_{12}' & \varepsilon_{13}' \\ \varepsilon_{21}' & \varepsilon_{22}' & \varepsilon_{23}' \\ \varepsilon_{31}' & \varepsilon_{32}' & \varepsilon_{33}' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta \varepsilon_{11} + \sin \theta \varepsilon_{21} & \cos \theta \varepsilon_{12} + \sin \theta \varepsilon_{22} & \cos \theta \varepsilon_{13} + \sin \theta \varepsilon_{23} \\ -\sin \theta \varepsilon_{11} + \cos \theta \varepsilon_{21} & -\sin \theta \varepsilon_{12} + \cos \theta \varepsilon_{22} & -\sin \theta \varepsilon_{13} + \cos \theta \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta \varepsilon_{11} + \sin 2\theta \varepsilon_{12} + \varepsilon_{22} \sin^2 \theta & -\cos \theta \varepsilon_{11} + \sin^2 \theta \varepsilon_{21} + \cos^2 \theta \varepsilon_{12} + \sin \theta \cos \theta \varepsilon_{22} & \cos \theta \varepsilon_{13} + \sin \theta \varepsilon_{23} \\ \sin^2 \theta \varepsilon_{11} + 2 \sin \theta \cos \theta \varepsilon_{12} + \cos^2 \theta \varepsilon_{22} & & -\sin \theta \varepsilon_{13} + \cos \theta \varepsilon_{23} \\ \text{sym} & & \varepsilon_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta \varepsilon_{11} + \sin 2\theta \varepsilon_{12} + \sin^2 \theta \varepsilon_{22} & -\frac{1}{2} \sin 2\theta \sin \theta (\varepsilon_{22} - \varepsilon_{11}) + \cos 2\theta \varepsilon_{12} & \cos \theta \varepsilon_{13} + \sin \theta \varepsilon_{23} \\ \text{sym} & \sin^2 \theta \varepsilon_{11} - \sin 2\theta \varepsilon_{12} + \cos^2 \theta \varepsilon_{22} & \cos \theta \varepsilon_{23} - \sin \theta \varepsilon_{31} \\ & & \varepsilon_{33} \end{bmatrix}$$

